Mean square stabilization of discrete-time switching Markov jump linear systems

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Mean Square Stabilization of Discrete-time Switching Markov Jump Linear Systems†

Haibo Qu[1] | Jialei Hu[1] | Yang Song*[1,2] | Taicheng Yang[3]

[1]Department of Automation, School of Mechatronic Engineering and Automation, Shanghai University, Shanghai, China
[2]Shanghai Key Laboratory of Power Station Automation Technology , Shanghai, China
[3]Dept. of Engineering and design, University of Sussex, Brighton, UK

Correspondence
*Yang Song , Shanghai University, Shanghai,China Email: y_song@shu.edu.cn

Present Address
Shanghai University, 149 Yanchang Road, Jing'an District, Shanghai

Summary

This paper consider a special class of hybrid system called switching Markov jump linear system. The system transition is governed by two rules. One is Markov chain and the other is a deterministic rule. Furthermore, the transition probability of the Markov chain is not only piecewise but also orchestrated by a deterministic switching rule. In this paper the mean square stability of the systems is studied when the deterministic switching is subject to two different dwell time conditions: having a lower bound and having both lower and high bounds. The main contributions of this paper are two relevant stability theorems for the systems under study. A numerical example is provided to demonstrate the theoretical results.

KEYWORDS:
Mean square stability, Deterministic switching, Markovain chain, Switching Markov jump linear system, Dwell time.

1 | INTRODUCTION

Markov jump linear system (MJLS) is a class of stochastic switched systems with wide applications. They are often used to model the dynamics of systems with random faults, unpredictable events, structural changes, networked control systems, etc.[1,2,34,5]Ṡeveral different notions of stability are defined respectively for stochastic systems, which are also applicable to MJLSs. These are δ-momentstability,[6] mean square stability (MS),[7,8] almost-sure stability(AS).[9,10,11] MS stability defines that the expectation of system state norm asymptotically converges to zero. This is an important special case of the δ-moment stability (δ=2). AS stability means that almost all realizations of system trajectory approaches to zero.[12] For MJLS, δ-moment stability implies AS stability, but not vice versa. For more results on the stability of MJLS, please refer to [13,14]. In addition, some new extension of stability results on semi-MJLS are also proposed.[15,16,17]

Recently, a new switched system is proposed in which the switching of subsystem is dominated jointly by a deterministic rule and stochastic rule. [11,18,19] The background of this new system is shown in FIGURE 1, where the multi-controllers switching and the plant modes switching coexist. Since that control strategy is generally designed previously by the engineers, thus the controller switching is deterministic here. On the other hand, note the fact that the changes of plant modes are often caused by unexpected factors, e.g. random failures, the switching of plant modes is supposed to be stochastic and is fit for a Markovain chain further. This new type of switched system is also called switching Markov jump linear system[11] . In a switching MJLS, the transition rate of the Markov chain can be fixed[22] or variable[11] , while the deterministic switching is generally subject to constraints on the dwell time[23] or the average dwell time.[24]

Below we outline some stability results relevant to the study in this paper. In [19], sufficient conditions are provided for the AS stability of continuous switching MJLSs where the Markov chain has a fixed transition rate. A further study on the Markov

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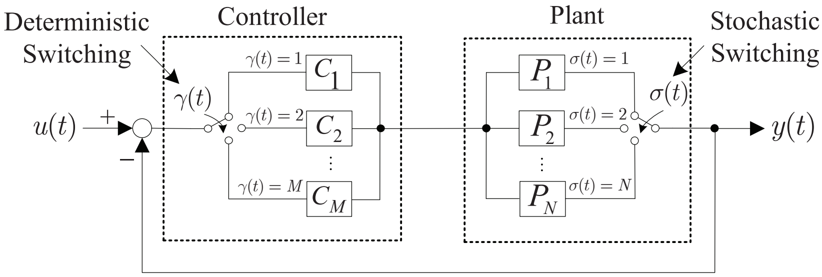

FIGURE 1 Background of Switching MJLS.

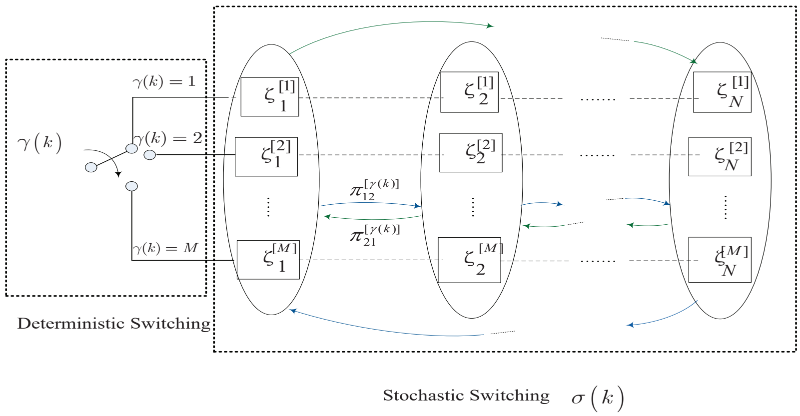

FIGURE 2 The structure of a SMJLS.

process extends the result to non-fix but piecewise transitions.[20] On the other hand, sufficient conditions for the MS stability are proved for continuous switching MJLSs subject to minimal dwell time.[18] Furthermore, a co-design of controller and stabilizing switching rule is presented in [21] which ensures the H_2 and H_∞ performance. Different from the above and as described in the abstract, we study the MS stability for a special class of MJLSs and have proved two sufficient conditions (Theorem 1 and 2 in section 3). In addition, the system performances are also analyzed.

The rest of this paper is organized as follows. The problem formulation, main results, a numerical example and conclusions are presented in sections 2, 3, 4 and 5 respectively.

Notations: 0_N(or1_N)is an$N \times N$dimension matrix with all the elements being 0 (or 1).R_+andZ_+denote the set of non-negative real numbers and set of non-negative integers, respectively;$diag\{...\}$stands for a block-diagonal matrix. In addition, symbol $\circ$ and $\otimes$ are used as the terms for the product of Hadamard and Kronecker .

2 | PROBLEM FORMULATION

Consider a discrete-time SMJLS:

$$x(k+1) = A^{[\gamma(k)]}_{\sigma(k)} x(k) \tag{1}$$

where the deterministic switching $\gamma(k) \in \mathcal{M} := \{1, 2, \ldots, M\}$is a piecewise function, the stochastic switching $\sigma(k) \in \mathcal{N} := \{1, 2, \ldots, N\}$is governed by a N-mode Markov chain with piecewise transition probability, i.e. the switching of $\gamma(k)$will bring the change of transition probability of $\sigma(k)$. The structure of a SMJLS is illustrated by FIGURE 2, where $\zeta^i_j : x(k+1) = A^{[i]}_j x(k)$.

When $\gamma(k) = j$, the one-step transition probability of the Markov chain $\sigma(k)$ at this instant is denoted by $\pi^{[j]}_{rs}$, where $\pi^{[j]}_{rs} := \Pr\{\sigma(k+1) = s | \sigma(k) = r, \gamma(k) = j\} \quad, r, s \in \mathcal{N}$. Matrix $\Pi^{[j]} = [\pi^{[j]}_{rs}]_{N\times N}$ is the transition probability matrix of Markov chain $\sigma(k)$ which is piecewise. In this paper, Markov chain $\sigma(k)$ is assumed to be irreducible for arbitrary transition probability

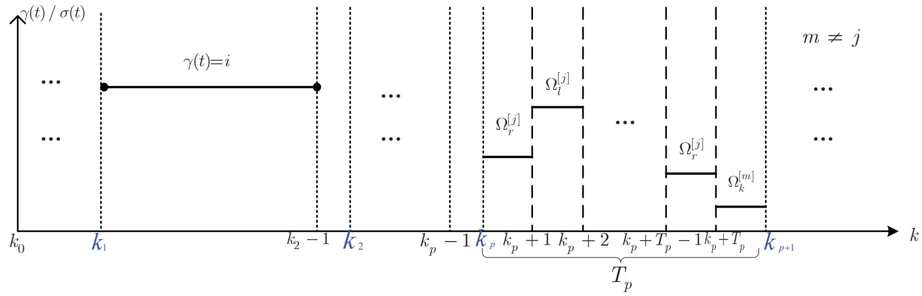

FIGURE 3 Switching Sequence.

matrix $\Pi^{[j]}$, hence the unique invariant distribution $\pi^{[j]} := [\pi_1^{[j]} \ \cdots \pi_N^{[j]}]$ exists and can be calculated out by $\pi^{[j]}\Pi^{[j]} = \pi^{[j]}$, $\sum_{i=1}^{N} \pi_i^{[j]} = 1, j \in \mathcal{M}$.

The initial conditions of MJLS (1) include: initial state x_0, initial deterministic switching position γ_0 and initial probability distribution, where $f^{[\gamma_0]} := [f_1^{[\gamma_0]} \ \cdots f_N^{[\gamma_0]}]$, $i \in \mathcal{N}$.

FGURE 3 illustrates the switching sequences of a discrete-time SMJLS, where $k_1, k_2, \cdots$ are the deterministic switching instants, k_p+1, k_p+2, $\cdots$ represent the stochastic switching instants during the interval $\Theta_p := [k_p, k_{p+1})$, subsystems $\Omega_r^{[j]}, \Omega_l^{[j]}, \cdots$ is actuated successively in interval Θ_p, the p-th dwell time T_p=k_{p+1}−k_p.

Definition1: The MJLS (1) is said to be mean square stable (MS-stable) if $\lim_{k\to\infty} E\left[\|x(k)\|^2\right] = 0$ for any initial condition x_0 and any initial probability distribution $f^{[\gamma_0]}$.

3 | MAIN RESULTS

This section presents sufficient conditions for the mean square stability of the MJLS (1) when the dwell-time of the deterministic switching $\gamma(k)$ is subject to constraints. The deterministic switching law is expressed as follows:

$$\gamma(k) = j \in M \qquad k \in \Theta_p := [k_p, k_{p+1}) \tag{2}$$

where k_p and k_{p+1} are the any two successive determined switching instants which satisfies

$$k_{p+1} - k_p \geq \Delta \geq 1 \tag{3}$$

Define a set of matrices:

$$\Psi_{D,h}^{[j]} := A_h^{T[j]} \left(\sum_{i_{D-1} \cdots, i_1, i_0}^{N} \pi_{h i_{D-1}} \prod_{l=1}^{D-1} \pi_{i_l i_{l-1}} A_{i_{D-1}}^{T[j]} \cdots A_{i_1}^{T[j]} P_{i_0}^{[k]} A_{i_1}^{[j]} \cdots A_{i_{D-1}}^{[j]} \right) A_h^{T[j]} \tag{4}$$

where $h \in \mathcal{N}, j \neq m \in \mathcal{M}$.

Theorem 1. SMJLS (1) with minimal dwell time constraint (3) is mean square stable, if there exists a set of definite matrices $P_i^{[j]}$, $i \in \mathcal{N}$, $j \in \mathcal{M}$ such that

$$\sum_{h=1}^{N} \pi_{ih} A_i^{T[j]} P_h^{[j]} A_i^{[j]} - P_i^{[j]} < 0 \qquad \forall i \in \mathcal{N}, \forall j \in \mathcal{M} \tag{5}$$

$$A_i^{T[j]} \left(\sum_{h=1}^{N} \pi_{ih} \Psi_{\Delta-1,h}^{[j]} \right) A_i^{[j]} - P_i^{[j]} < 0 \qquad \forall j \in \mathcal{M} \tag{6}$$

Proof: Construct piecewise Lyapunov function $V(k) = x(k) P_{\sigma(k)}^{[\gamma(k)]} x(k)$. Denote k_p and k_{p+1} as two successive determined switching respectively. Divide the dwell time Θ_p into two parts,$\Theta_p = \Theta_p^1 \cup \Theta_p^2$, $\Theta_p^1 = \left[k_p, k_{p+1} - \Delta\right)$, $\Theta_p^2 = \left[k_{p+1} - \Delta, k_{p+1}\right)$. Assume $\gamma(k) = j$, $\forall k \in \Theta_p$, and $\gamma(k_{p+1}) = m$, $m \neq j$.

When $k \in \Theta_p^1$, it can be seen from (5) that

$$\begin{aligned} & E\left[V(k+1)\mid\sigma(k)=i,\gamma(k)=j\right]-E[V(k)]=\sum_{h=1}^{N}\pi_{ih}x^T(k)A^{T[j]}_{i}P_h^{[j]}A_i^{[j]}x(k)-x^T(k)P_i^{[j]}x(k) \\ & =x^T(k)\left(\sum_{h=1}^{N}\pi_{ih}A^{T[j]}_{i}P_h^{[j]}A_i^{[j]}-P_i^{[j]}\right)x(k) \\ & <0 \end{aligned} \tag{7}$$

By letting $k=k_p,k_p+1,\cdots,k_{p+1}-1-\Delta$ in (7) ,it can be seen from that

$$E\left[V\left(k_{p+1}-\Delta\right)\right]<\mathrm{E}\left[V\left(k_p\right)\right] \tag{8}$$

For $k \in \Theta_p^2$, denote $q=k_{p+1}$, then

$$\begin{aligned} & E\left[V\left(k_{p+1}\right)\right]=E\left[V(q)\right]=E\left[x^T(q)P_{\sigma(q)}^{[m]}x(q)\right] \\ & =E_{\sigma(q-1)}\left[E\left[x^T(q)P_{\sigma(q)}^{[m]}x(q)|\sigma(q-1)=i_1\right]\right] \\ & =E\left[x^T(q-1)\sum_{i_0=1}^{N}\pi_{i_1i_0}A^{T[j]}_{i_1}P_{i_0}^{[m]}A_{i_1}^{[j]}x(q-1)\right] \\ & =E_{\sigma(q-2)}\left[E\left[x^T(q-1)\sum_{i_0=1}^{N}\pi_{i_1i_0}A^{T[j]}_{i_1}P_{i_0}^{[m]}A_{i_1}^{[j]}x(q-1)\middle|\sigma(q-2)=i_2\right]\right] \\ & =E\left[x^T(q-2)\sum_{i_1=1}^{N}\pi_{i_2i_1}\left(\sum_{i_0=1}^{N}\pi_{i_1i_0}A^{T[j]}_{i_2}A^{T[j]}_{i_1}P_{i_0}^{[m]}A_{i_1}^{[j]}A_{i_2}^{[j]}\right)x(q-2)\right] \\ & =E\left[x^T(q-2)\sum_{i_1=1 i_0=1}^{N}\pi_{i_2i_1}\pi_{i_1i_0}A^{T[j]}_{i_2}A^{T[j]}_{i_1}P_{i_0}^{[m]}A_{i_1}^{[j]}A_{i_2}^{[j]}x(q-2)\right] \\ & =\cdots \\ & =E_{\sigma(q-\Delta)}\left[x^T(q-\Delta)A^{T[j]}_{i_\Delta}\left(\sum_{i_{\Delta-1}\cdots,i_1,i_0=1}^{N}\pi_{i_\Delta i_{\Delta-1}}\prod_{l=1}^{\Delta-1}\pi_{i_li_{l-1}}A^{T[j]}_{i_{\Delta-1}}\cdots A^{T[j]}_{i_1}P_{i_0}^{[m]}A_{i_1}^{[j]}\cdots A_{i_{\Delta-1}}^{[j]}\right)A_{i_\Delta}^{[j]}x(q-\Delta)\right] \end{aligned} \tag{9}$$

It follows from (6) that

$$\begin{aligned} & E\left[V\left(k_{p+1}\right)\right]=E_{\sigma(q-\Delta)}\left[x^T(q-\Delta)\sum_{i_{\Delta-1}=1}^{N}\pi_{i_\Delta i_{\Delta-1}}A^{T[j]}_{i_\Delta}\Psi^{[j]}_{\Delta-1,i_{\Delta-1}}A_{i_\Delta}^{[j]}x(q-\Delta)\right] \\ & <E\left[x^T(q-\Delta)P_{\sigma(q-\Delta)}^{[j]}x(q-\Delta)\right] \\ & <E\left[V\left(k_{p+1}-\Delta\right)\right] \end{aligned} \tag{10}$$

Hence with (8) and (10), one can see that

$$E\left[V\left(k_{p+1}\right)\right]<E\left[V\left(k_p\right)\right] \tag{11}$$

From inequality (11), it can be seen that the Lyapunov function of the system is substantially reduced, and the system trajectory eventually converges to the equilibrium point. Then by Definition 1 and, the SMJLS (1) is MS-stable.

Theorem 1 considers the case that the deterministic switching subject to the constraints of minimal dwell time. Furthermore, the following theorem deals with the case that the dwell time has both lower bound and upper bound, i.e. $\nabla \geq k_{p+1}-k_p \geq \Delta$ for all $p \in \mathrm{N}$.

Theorem 2. Denote $\nabla \geq \Delta > 0$ as the upper and lower bound of deterministic switching, $\Xi := \sum_{i_0=1}^{N}\Phi_{i_0} \geq 0$. Define

$$R_h^{[j]} := \sum_{z=1}^{\nabla-1} A^{T}{}_h^{[j]} \left(\sum_{i_{z-1},\cdots,i_1,i_0=1}^{N} \pi_{hi_{z-1}} \prod_{l=1}^{z-1} \pi_{i_l i_{l-1}} A^{T}{}_{i_{z-1}}^{[j]} \cdots A^{T}{}_{i_1}^{[j]} \Phi_{i_0} A_{i_1}^{[j]} \cdots A_{i_{z-1}}^{[j]} \right) A_h^{[j]} \quad j \in \mathcal{M}, h \in \mathcal{N} \tag{12}$$

If there exists a set of matrices $P_i^{[j]} > 0,\ i \in \mathcal{N}\ , j \in \mathcal{M}$ such that

$$\sum_{h=1}^{N} \pi_{ih} A^{T}{}_i^{[j]} P_h^{[j]} A_i^{[j]} - P_i^{[j]} + \Xi < 0 \qquad \forall i, h \in \mathcal{N} \ \ \forall j \in \mathcal{M} \tag{13}$$

$$A^{T}{}_i^{[j]} \left(\sum_{h=1}^{N} \pi_{ih} \Psi_{\Delta-1,h}^{[j]} \right) A_i^{[j]} - P_i^{[j]} + R_i^{[j]} < 0 \qquad \forall i \in \mathcal{N} \ \ \forall j \in \mathcal{M} \tag{14}$$

SMJLS (1) is mean square stable, and

$$\sum_{k=0}^{\infty} \mathrm{E}\left[x^T(k)\Xi x(k)\right] < x^T(0) P_{\sigma(0)}^{\gamma(0)} x(0)$$

Proof: Due to $\Xi \geq 0$, we have $\Phi_{i_0} \geq 0$, hence matrices $R_i^{[j]}$ defined in (12) are positive semidefinite, then (13) and (14) implies that the conditions (5) and (6) are satisfied. Hence system (1) is mean square stable. In addition, from the definition (11) and the inequalities (13) and (14), one can see that $P_i^{[j]} > R_i^{[j]}$ and

$$\begin{aligned}
&\sum_{h=1}^{N} \pi_{ih} A^{T}{}_i^{[j]} (P_h^{[j]} - R_h^{[j]}) A_i^{[j]} - (P_h^{[j]} - R_h^{[j]}) < -\Xi - \sum_{h=1}^{N} \pi_{ih} A^{T}{}_i^{[j]} (R_h^{[j]}) A_i^{[j]} + R_h^{[j]} \\
&< -A^{T}{}_h^{[j]} \left(\sum_{i_{\nabla-1},\cdots,i_1,i_0=1}^{N} \pi_{hi_{\nabla-1}} \prod_{l=1}^{\nabla} \pi_{i_l i_{l-1}} A^{T}{}_{i_{\nabla-1}}^{[j]} \cdots A^{T}{}_{i_1}^{[j]} O_{i_0} A_{i_1}^{[j]} \cdots A_{i_{\nabla-1}}^{[j]} \right) A^{T}{}_h^{[j]} \\
&< 0
\end{aligned} \tag{15}$$

Noticing that $k_{p+1} - k_p \geq \Delta \geq 1$ and Eq. (10), it can be obtained that

$$\begin{aligned}
&E\left[V\left(k_{p+1}\right)\right] = E_{\sigma(q-\Delta)}\left[x^T(q-\Delta) \sum_{i_{\Delta-1}=1}^{N} \pi_{i_\Delta i_{\Delta-1}} A^{T}{}_{i_\Delta}^{[j]} \Psi_{\Delta-1,i_{\Delta-1}}^{[j]} A_{i_\Delta}^{[j]} x(q-\Delta)\right] \\
&< E\left[x^T(q-\Delta)(P_{\sigma(q-\Delta)}^{[j]} - R_{\sigma(q-\Delta)}^{[j]}) x(q-\Delta)\right] \\
&< E\left[V\left(k_{p+1}-\Delta\right)\right] - E\left[x^T(q-\Delta) R_{\sigma(q-\Delta)}^{[j]} x(q-\Delta)\right]
\end{aligned} \tag{16}$$

From the inequality (8), one can see $E\left[V\left(k_{p+1}\right)\right] < E\left[V\left(k_p\right)\right] - E\left[x^T(q-\Delta) R_{\sigma(q-\Delta)}^{[j]} x(q-\Delta)\right]$. By summing up for all $p \in \mathrm{N}$ and taking into account that $k_{p+1} - k_p \leq \nabla$, it yields

$$\begin{aligned}
&\sum_{k=0}^{\infty} \mathrm{E}\left[x^T(k)\Xi x(k)\right] = \sum_{p=0}^{\infty} E_{\sigma(k_p)}[x^T(k_p) \sum_{r=1}^{\nabla-1} A^{T}{}_{\sigma(k_p)}^{[j]} [\sum_{\sigma(k_p)-1,\cdots,\sigma(k_{p+1})-1=1}^{N} \prod_{l=\Delta}^{k_{p+1}-k_p} \pi_{i_l i_{l-1}} A^{T}{}_{\sigma(k_p+1)}^{[j]} \\
&\cdots A^{T}{}_{\sigma(k_{p+1}-1)}^{[j]} O_{\sigma(k_{p+1})} A_{\sigma(k_{p+1}-1)}^{[j]} \cdots A_{\sigma(k_p+1)}^{[j]}] A_{\sigma(k_p)}^{[j]} x(k_p)] \\
&\leq \sum_{p=0}^{\infty} E_{\sigma(k_p)}\left[x^T(k_p) R_{\nabla,\sigma(k_p)}^{[j]} x(k_p)\right] \\
&< V\left(x\left(k_0\right)\right)
\end{aligned} \tag{17}$$

This completes the proof. To simplify the expression of Theorem 3, the following lemma is presented which could describe Theorem 3 in iteration form.

Lemma 1. Given matrix $Q_r \in R^{N^2 \times N^2}$ and a set of matrices $B_i, P_i, C_i \in R^{N \times N}$, $i \in \mathcal{N}$. Define

$$\Omega_{r,q} := B_q \left(\sum_{i_{r-1} \cdots, i_1, i_0 = 1}^{N} \pi_{q i_{r-1}} \prod_{l=1}^{r-1} \pi_{i_l i_{l-1}} B_{i_{r-1}} \cdots B_{i_0} P_{i_0} C_{i_0} \cdots C_{i_{r-1}} \right) C_q \tag{18}$$

where $q \in \mathcal{N}$, $r \in Z_+$, then, there exists that

$$diag\left\{ \Omega_{\mathrm{r},1}, \Omega_{\mathrm{r},2}, ..., \Omega_{\mathrm{r},N} \right\} = Q_r \circ 1_{N^2} \tag{19}$$

where

$$Q_r = \left(\left(B Q_{r-1} C \right) \circ 1_{N^2} \right) \left(1_N \otimes I_N \right) \tag{20}$$

$$Q_0 = diag\left\{ P_1, P_2, ..., P_N \right\} \left(1_N \otimes I_N \right) \tag{21}$$

$$B = diag\left\{ B_1, B_2, ..., B_N \right\} \left(\Pi \otimes I_N \right) \tag{22}$$

$$C = diag\left\{ C_1, C_2, ..., C_N \right\} \left(1_N \otimes I_N \right) \tag{23}$$

$$1_{N^2} = diag\left\{ 1_N, 1_N, ..., 1_N \right\} \tag{24}$$

$\Pi = \left[\pi_{ij} \right]$ is the transition matrix.

Proof: From (14), one can obtain

$$\Omega_{k,i_k} = B_{i_k} \left(\sum_{i_{k-1}=1}^{N} \pi_{i_k i_{k-1}} \Omega_{k-1, i_{k-1}} \right) C_{i_k} \quad i_k \in \mathcal{N} \tag{25}$$

Based on (20) -(23), it follows

$$\begin{aligned}
Q_1 &= \left(\left(B Q_0 C \right) \circ 1_{N^2} \right) \left(1_N \otimes I_N \right) \\
&= \left(\left(\left(\begin{pmatrix} \pi_{11} B_1 & \pi_{12} B_1 & ... & \pi_{1N} B_1 \\ \pi_{21} B_2 & \cdots & \cdots & \pi_{2N} B_2 \\ ... & ... & \ddots & ... \\ \pi_{N1} B_N & \pi_{N2} B_N & ... & \pi_{NN} B_N \end{pmatrix} \begin{pmatrix} P_1 & P_1 & ... & P_1 \\ P_2 & \cdots & \cdots & P_2 \\ ... & ... & \ddots & ... \\ P_N & P_N & ... & P_N \end{pmatrix} \begin{pmatrix} C_1 & & ... & \\ & C_2 & \cdots & \\ ... & ... & \ddots & ... \\ & & ... & C_N \end{pmatrix} \right) \circ \begin{pmatrix} 1_N & 0 & ... & 0 \\ 0 & 1_N & \cdots & 0 \\ ... & ... & \ddots & ... \\ 0 & 0 & ... & 1_N \end{pmatrix} \right) \right) \times \left(1_N \otimes I_N \right) \\
&= \left(\left(\left(\begin{pmatrix} \pi_{11} B_1 & \pi_{12} B_1 & ... & \pi_{1N} B_1 \\ \pi_{21} B_2 & \cdots & \cdots & \pi_{2N} B_2 \\ ... & ... & \ddots & ... \\ \pi_{N1} B_N & \pi_{N2} B_N & ... & \pi_{NN} B_N \end{pmatrix} \begin{pmatrix} P_1 C_1 & P_1 C_2 & ... & P_1 C_N \\ P_2 C_1 & \cdots & \cdots & P_2 C_N \\ ... & ... & \ddots & ... \\ P_N C_1 & P_N C_2 & ... & P_N C_N \end{pmatrix} \right) \right) \circ \begin{pmatrix} 1_N & 0 & ... & 0 \\ 0 & 1_N & \cdots & 0 \\ ... & ... & \ddots & ... \\ 0 & 0 & ... & 1_N \end{pmatrix} \right) \left(1_N \otimes I_N \right) \\
&= \begin{pmatrix} \sum_{j=1}^{N} \pi_{1j} B_1 P_j C_1 & * & ... & * \\ * & \sum_{j=1}^{N} \pi_{2j} B_2 P_j C_2 & ... & * \\ ... & ... & \ddots & ... \\ * & * & ... & \sum_{j=1}^{N} \pi_{Nj} B_N P_j C_N \end{pmatrix} \circ \begin{pmatrix} 1_N & 0 & ... & 0 \\ 0 & 1_N & ... & 0 \\ ... & ... & \ddots & ... \\ 0 & 0 & ... & 1_N \end{pmatrix} \times \left(1_N \otimes I_N \right)
\end{aligned} \tag{26}$$

$$= \begin{pmatrix} \Omega_{1,1} & 0 & ... & 0 \\ 0 & \Omega_{1,2} & ... & 0 \\ ... & ... & \ddots & ... \\ 0 & 0 & ... & \Omega_{1,N} \end{pmatrix} \times \left(1_N \otimes I_N\right)$$

$$= \begin{pmatrix} \Omega_{1,1} & \Omega_{1,1} & ... & \Omega_{1,1} \\ \Omega_{1,2} & \cdots & \cdots & \Omega_{1,2} \\ ... & ... & \ddots & ... \\ \Omega_{1,N} & \Omega_{1,N} & ... & \Omega_{1,N} \end{pmatrix}$$

Moreover, taking the iterative way, we can obtain that

$$Q_r = \left(\left(BQ_{r-1}C\right) \circ 1_{N^2}\right)\left(1_N \otimes I_N\right)$$

$$= \left(\left(\begin{pmatrix} \pi_{11}B_1 & \pi_{12}B_1 & ... & \pi_{1N}B_1 \\ \pi_{21}B_2 & \cdots & ... & \pi_{2N}B_2 \\ ... & ... & \ddots & ... \\ \pi_{N1}B_N & \pi_{N2}B_N & ... & \pi_{NN}B_N \end{pmatrix}\begin{pmatrix} \Omega_{T-1,1} & \Omega_{T-1,1} & ... & \Omega_{T-1,1} \\ \Omega_{T-1,2} & \cdots & ... & \Omega_{T-1,2} \\ ... & ... & \ddots & ... \\ \Omega_{T-1,N} & \Omega_{T-1,N} & ... & \Omega_{T-1,N} \end{pmatrix}\begin{pmatrix} C_1 & & ... & \\ & C_2 & ... & \\ ... & ... & \ddots & ... \\ & & ... & C_N \end{pmatrix}\right) \circ \begin{pmatrix} 1_N & 0 & ... & 0 \\ 0 & 1_N & ... & 0 \\ ... & ... & \ddots & ... \\ 0 & 0 & ... & 1_N \end{pmatrix}\right) \times \left(1_N \otimes I_N\right) \tag{27}$$

$$= \begin{pmatrix} \Omega_{T,1} & \Omega_{T,1} & ... & \Omega_{T,1} \\ \Omega_{T,2} & \cdots & ... & \Omega_{T,2} \\ ... & ... & \ddots & ... \\ \Omega_{T,N} & \Omega_{T,N} & ... & \Omega_{T,N} \end{pmatrix}$$

Therefore, we have

$$Q_r \circ \begin{pmatrix} 1_N & 0 & ... & 0 \\ 0 & 1_N & ... & 0 \\ ... & ... & \ddots & ... \\ 0 & 0 & ... & 1_N \end{pmatrix} = \begin{pmatrix} \Omega_{r,1} & 0 & ... & 0 \\ 0 & \Omega_{r,2} & ... & 0 \\ ... & ... & \ddots & ... \\ 0 & 0 & ... & \Omega_{r,N} \end{pmatrix} \tag{28}$$

Hence

$$diag\left\{\Omega_{\mathrm{r},1}, \Omega_{\mathrm{r},2}, ..., \Omega_{\mathrm{r},N}\right\} = Q_r \circ 1_{N^2}$$

This completes the proof. By lemma 1, theorem 2 can be rewritten as below.

Theorem 3. ∇, Δ, Ξ is defined as in Theorem 2, semi-definite matrices

$$R^{[j]} := \sum_{z=0}^{\nabla}\left(W_z^{[j]} \circ 1_{N^2}\right) \ , \ j \in \mathcal{M} \tag{29}$$

If there exists a set of $P_i^{[j]} > 0$, $i \in \mathcal{N}$, $j \in \mathcal{M}$ such that

$$Q_1^{[j]} \circ 1_{N^2} - diag\left\{P_1^{[j]} - \Xi, P_2^{[j]} - \Xi, ..., P_N^{[j]} - \Xi\right\} < 0 \quad i \in \mathcal{N} \ \ j \in \mathcal{M} \tag{30}$$

$$Q_\Delta^{[m]} \circ 1_{N^2} - diag\left\{P_1^{[j]}, P_2^{[j]}, ..., P_N^{[j]}\right\} + R^{[j]} < 0 \qquad \forall m \neq j \in \mathcal{M} \tag{31}$$

then SMJLS (1) mean square stable with the dwell time constraint of $\nabla \geq k_{p+1} - k_p \geq \Delta$ where

$$Q_T^{[m]} = \left(\left(BQ_{T-1}^{[m]}C\right) \circ 1_{N^2}\right)\left(1_N \otimes I_N\right) \tag{32}$$

$$W_T^{[j]} = \left(\left(BW_{T-1}^{[j]}C\right) \circ 1_{N^2}\right)\left(1_N \otimes I_N\right) \tag{33}$$

$$B = diag\left\{A^T{}_1^{[j]}, A^T{}_2^{[j]}, A^T{}_3^{[j]}, ...A^T{}_N^{[j]}\right\}\left(\Pi^{[j]} \otimes I_N\right) \tag{34}$$

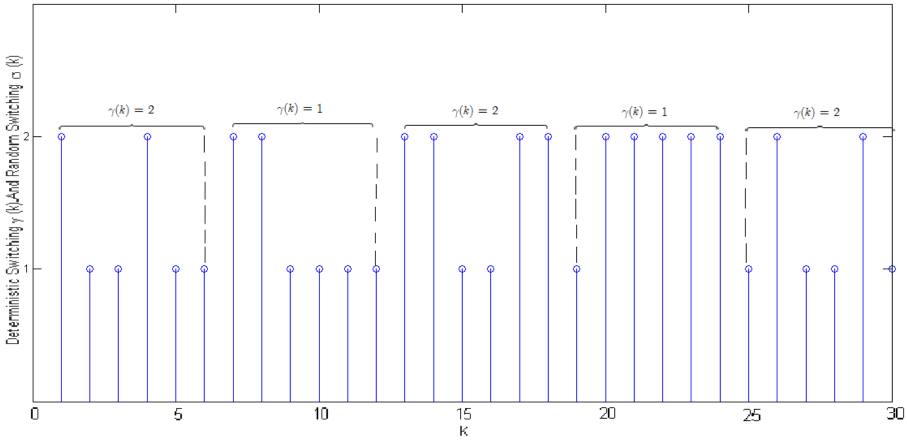

FIGURE 4 Switching signal.

$$Q_0^{[m]} = diag\left\{P_1^{[m]}, P_2^{[m]}, ..., P_N^{[m]}\right\}\left(1_N \otimes I_N\right) \tag{35}$$

$$W_0^{[j]} = diag\left\{O_1, O_2, ..., O_N\right\}\left(1_N \otimes I_N\right) \tag{36}$$

$$C = diag\left\{A^{T[j]}_{1}, A^{T[j]}_{2}, ..., A^{T[j]}_{N}\right\}\left(1_N \otimes I_N\right) \tag{37}$$

Proof: Based on Lemma 1, it is easy to see that inequalities (30) and (31) are equivalent to inequalities (13) and (14). Then following similiar proof line in Theorem, the conclusion of this theorem can be obtained.

4 | NUMERICAL EXAMPLE

Considering the discrete time SMJLS:

$$x(k+1) = A_{\sigma(k)}^{[\gamma(k)]} x(k)$$

Where $\gamma(k) = 1, 2$, $\sigma(k) = 1, 2$, the parameters are as follows

$$A_1^{[1]} = \begin{bmatrix} 0.6 & 0.8 \\ 0 & 1 \end{bmatrix} A_2^{[1]} = \begin{bmatrix} 0 & -0.8 \\ 0.8 & 0 \end{bmatrix} A_1^{[2]} = \begin{bmatrix} 0.8 & 0.6 \\ 0 & 1 \end{bmatrix} A_2^{[2]} = \begin{bmatrix} 0 & 0.1 \\ -0.1 & 1.2 \end{bmatrix}$$

The one-step transition probability matrices of Markov chains are

$$\Pi^{[1]} = \begin{bmatrix} 0.5 & 0.5 \\ 0.7 & 0.3 \end{bmatrix} \Pi^{[2]} = \begin{bmatrix} 0.2 & 0.8 \\ 0.4 & 0.6 \end{bmatrix}$$

the corresponding stationary distribution are $\pi^{[1]} = [3/7 \;\; 4/7]$,$\pi^{[2]} = [7/12 \;\; 5/12]$. Applying linear matrix inequality toolbox to solve inequalities (13)-(14) of theorem 2. Define $\Xi := \sum_{i_0=1}^{N} \Phi_{i_0} \geq 0$ in the theorem 2, one can obtain that

$$P_1^{[1]} = \begin{bmatrix} -2.3301 & 1.2254 \\ 1.2254 & 4.9010 \end{bmatrix} P_2^{[1]} = \begin{bmatrix} 0.0731 & -0.9410 \\ -0.9410 & -2.2842 \end{bmatrix}$$
$$P_1^{[2]} = \begin{bmatrix} -3.3691 & -1.4475 \\ -1.4475 & 6.2444 \end{bmatrix} P_2^{[2]} = \begin{bmatrix} -2.3993 & -1.3551 \\ -1.3551 & 11.7880 \end{bmatrix}$$

FIGURE 4 shows the switching laws of deterministic switching and stochastic switching, the dwell time of deterministic switching is equal to 6. And the FIGURE 5 shows the trajectory of$x(k)^T x(k)$ with the initial condition $x_0 = \begin{bmatrix} 10 & 9 \end{bmatrix}^T$. As shown in FIGURE 5, the trajectory of $x(k)^T x(k)$ converges to zero, hence the system is mean square stable.

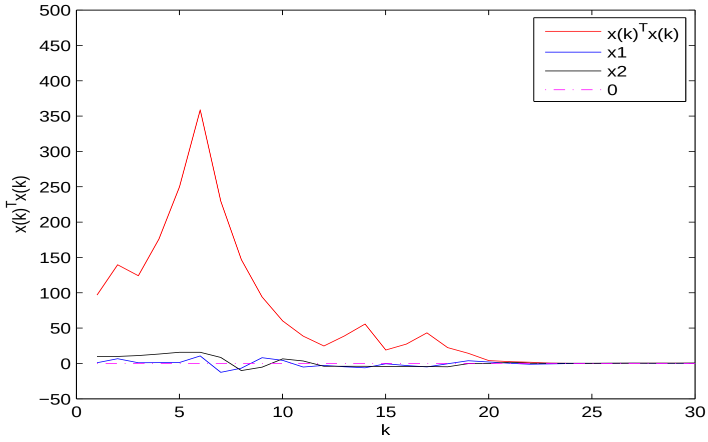

FIGURE 5 The trajectory of $x(k)^T x(k)$.

5 | CONCLUSION

This paper deals with the mean-square stability of discrete-time switching Markov jump linear system which is simultaneously subject to a deterministic switching signal and a Markov switching signal. Sufficient conditions for mean-square stability of the SMJLS are proposed respectively for the two cases, i.e. the dwell time of the deterministic switching only has lower bound as well as that the dwell time had both upper and lower bounds. Besides, for the latter case, a constraint on evaluating system performance is also presented accompanied with the stability conditions. Another expression of the main results which is more concise are also provided. Finally, a numerical example is given to demonstrates the effectiveness of the proposed results in this paper.

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Author contributions

The main contributions of this paper are two relevant stability theorems for the systems under study.

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None.

Conflict of interest

The authors declare no potential conflict of interests.

SUPPORTING INFORMATION

None.

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AUTHOR BIOGRAPHY

Yang Song. Dr. Yang Song, Professor, received his B.S and Ph.D in Electrical Engineering from the Department of Automation, Nanjing University of Science and Technology in 1998 and 2006. He worked as a Post-doctor in the Department of Automation, School of Mechatronics Engineering and Automation, Shanghai University from 2006 to 2008. He is now with Shanghai University. From July 2012 to July 2013, he was a visiting scholar with the University of Sussex, Brighton, UK. He is a member of Data Driven Control and Learning and Optimization Special Committee of Chinese Association of Automation, and is also a member of Shanghai Association for System Simulation. His research interests are networked control system, distributed control, switched systems and intelligent control.

Haibo Qu Dr. Haibo Qu, Postgraduate, received his B.D in Electrical Engineering from the Department of Automation, Qilu University of Technology in 2017. In the same year, he was awarded as an outstanding graduate, his graduation design, i.e. design of mechanical arm stamping system based on singlechip ,has also been rated as excellent graduation design. He is master student at the School of Mechatronics Engineering and Automation, Shanghai University. His main research interests are switched system, networked control system.

Jialei Hu. Dr. Jialei Hu, received his B.D from the institute of Automation, Zhejiang Institute of Technology in 2015. He studied at the Leibniz University in Hanover, Germany as an exchange student in 2016. He received his B.S from the Department of Automation, School of Mechanical and Electrical Engineering and Automation, Shanghai University in 2017. His main research interests are switched system, hybrid system and networked control system .

Taicheng Yang. Dr. Tai Yang obtained his PhD degree at the control system centre, UMIST in 1987. He joined the University of Sussex in 1990 as a Lecturer and became a Reader in Computer and Control Engineering in 1999. He had ten years of industrial experience before his academic career. His current research interests are: Networks and control, Power system control and wind power generation, and Control theory and applications in general.

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